MATH 235: Computational Problems

Chapter 6: Eigenvectors and Diagonalization

* **Given a linear mapping, a basis, and [x]B: Find the B-matrix of L and determine [L[x]]B**
  + Augment [B|L], RREF B and L is [L]B
  + [L[x]]B = [L]B [x]B
* **Determine which of the given vectors are eigenvectors of a given matrix A**
  + For each vector v, see if Av is a scalar multiple of v
    - The scalar should be the eigenvalue since Av = λv
* **Find the eigenvalues of A and the basis for each eigenspace**
  + Solve for 0 = det(A – λI)
  + For each λ, find the matrix A – λI and solve the system
  + The basis vectors for the eigenspace are the eigenvectors
    - Can be found by negating all columns without leading ones and placing a one in entry i, where i is the column #
* **Find the algebraic and geometric multiplicity of all eigenvalues of A**
  + Solve for 0 = det(A – λI)
  + Algebraic multiplicity is the degree of each λ
  + Geometric multiplicity is the number of basis vectors for each eigenspace
* **Show that A is diagonalizable and find invertible matrix P and diagonal matrix D such that P-1AP = D**
  + Solve for 0 = det(A – λI)
  + A is diagonalizable iff g = a for all λ
  + P is the matrix with columns formed by the eigenvectors of A
  + D = diag(λ1, …, λn)
* **Show that Ay = B, where y is a large number**
  + Find P and D for A
  + Find P-1
  + Show that Ay = P Dy P-1